Comparing functional programs, or how to put λ-terms back in order

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E.W. Beth Dissertation Award, August 7th 2026

Theoretical Computer Science → Programming Languages

Comparing functional programs, or how to put λ-terms back in order

Studying functional programming languages

```
let sort (compare :  'a -> 'a -> bool) :  list 'a -> list 'a := ...
```

Investigating the problem of program equivalence

Are `quick_sort` and `bubble_sort` equivalent?

In Practice, program equivalence is used to verify the correctness of the changes made by the user or the compiler.

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Order(s) on λ-terms

How (in what **order**) does the computer compute syntactic programs?
$\rightarrow$ computation dynamics: operational semantics, evaluation strategies, rewriting

Program **Preorders** instead of Equivalences!
$\rightarrow$ Updating code instead of always writing equivalent programs

Improvement Theory: computation **costs** of programs
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Evaluation Order

My program

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let double (x :  int) :  int := x + x.
let my_computation := double(2+3).
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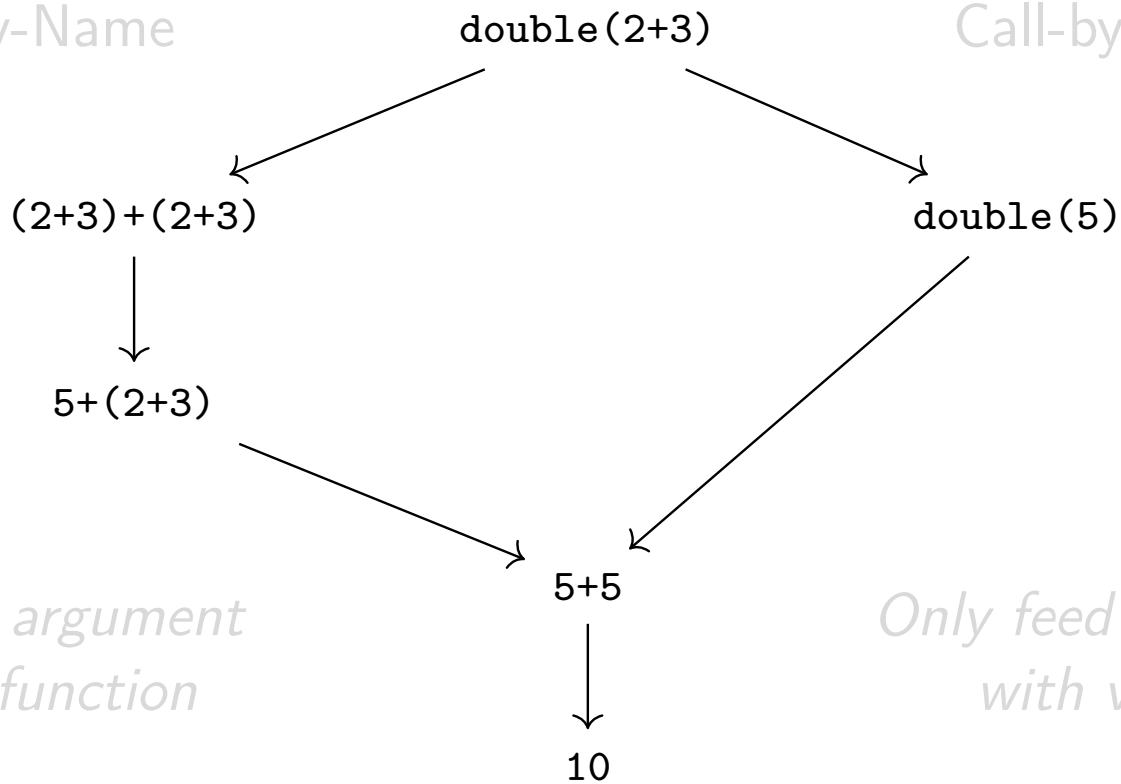

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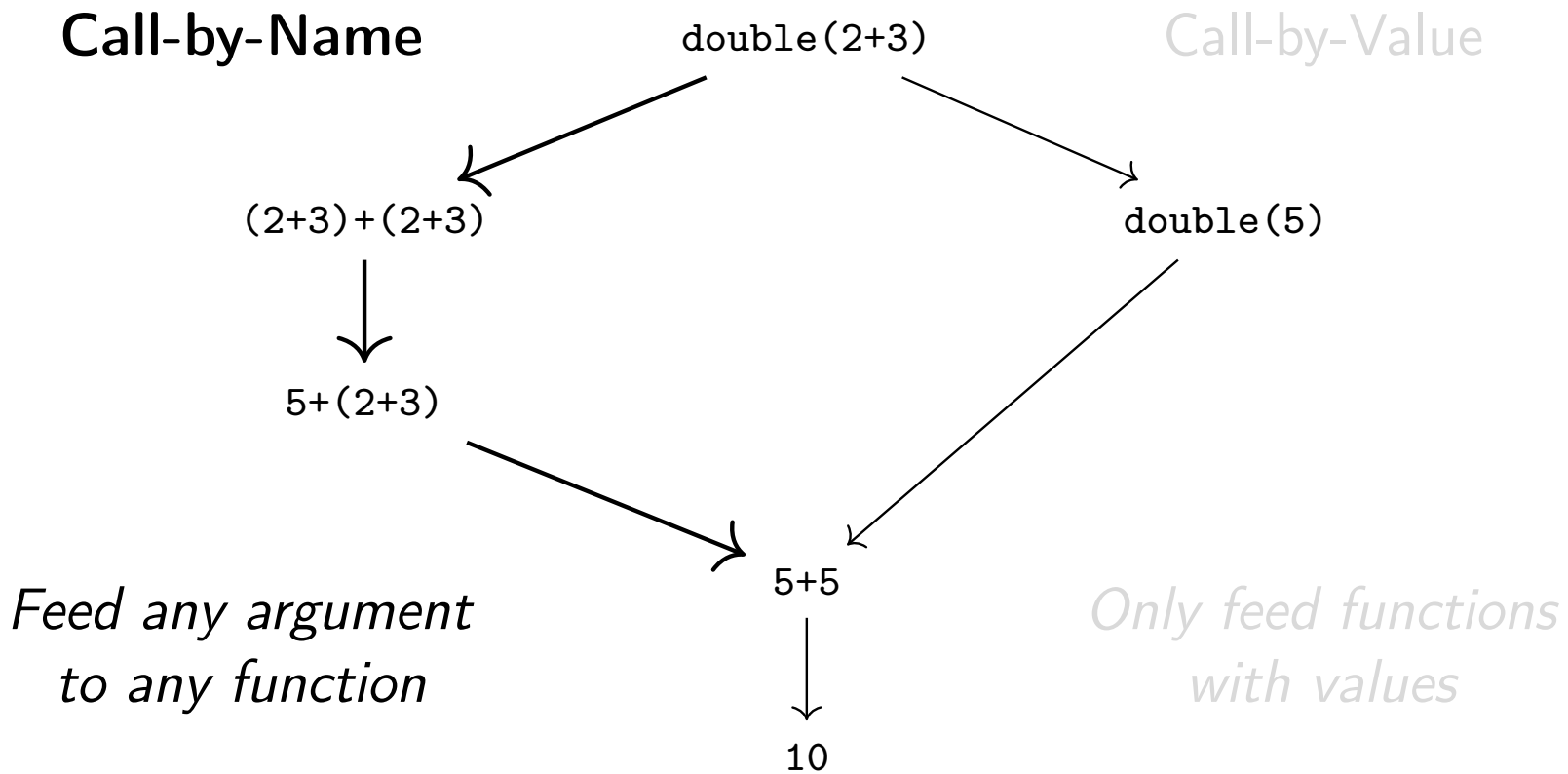

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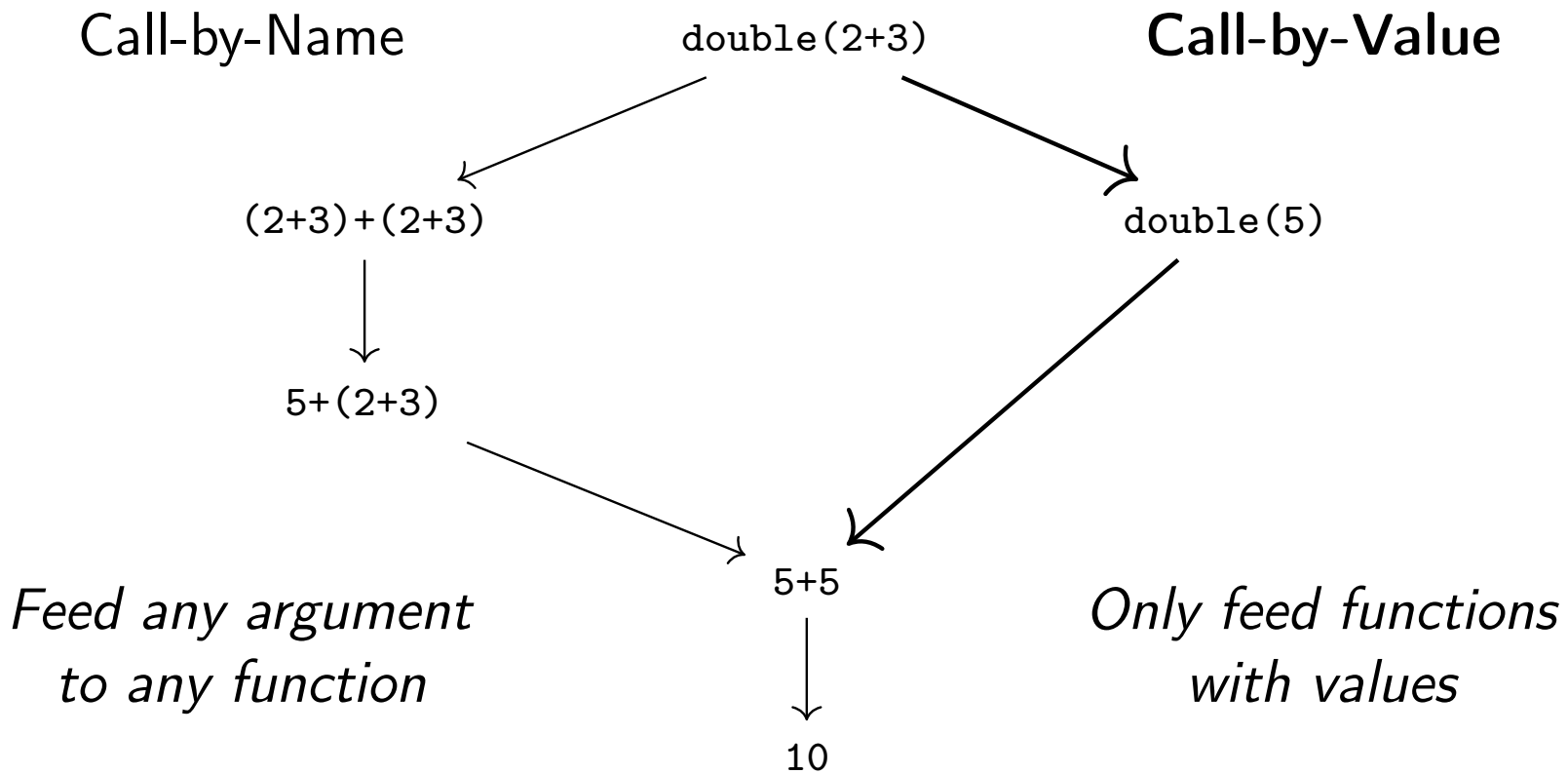

Computational Equivalence

Two coders write a program, how do we know if they are the same?

- **Inspecting the Code:**

A program

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let my_function :=
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The code is identical.

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Another program

```
let my_function :=
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The code is computationally equivalent.

Contextual Equivalence, Morris [1968]

- **Using the Programs:**

An execution environment

```
let n := 42.  ...
```

A program

```
let my_fun x
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              x+x.
```

The same execution environment

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let n := 42.  ...
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Another program

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let my_fun x
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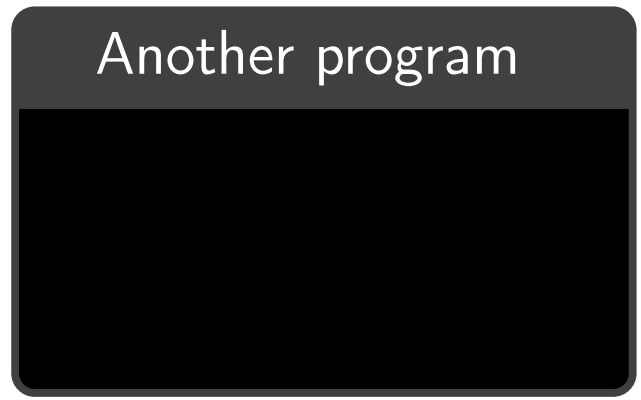

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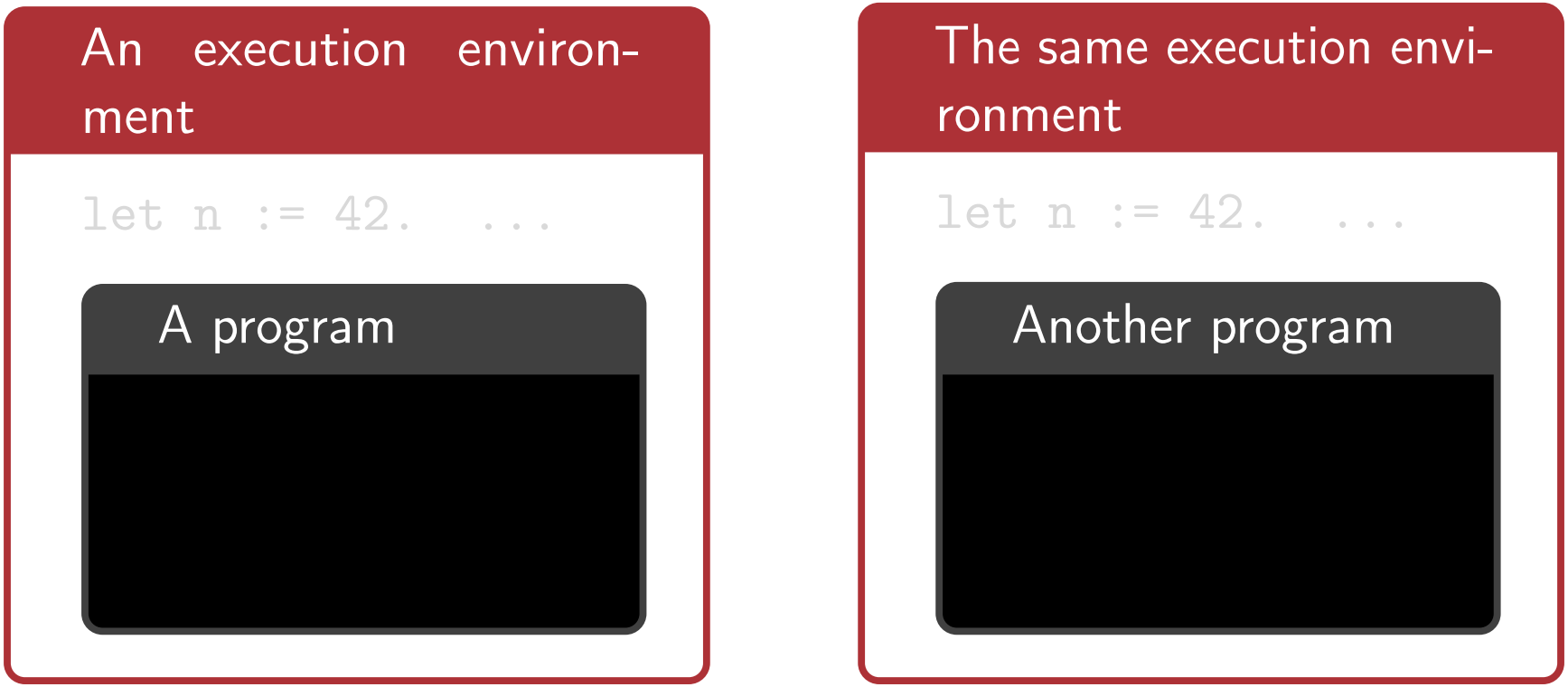

The code is tested.

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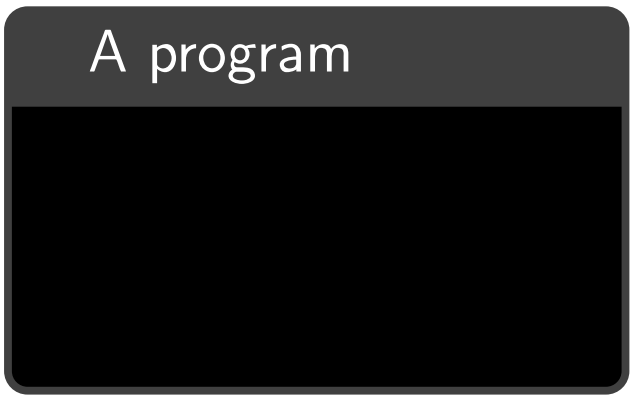

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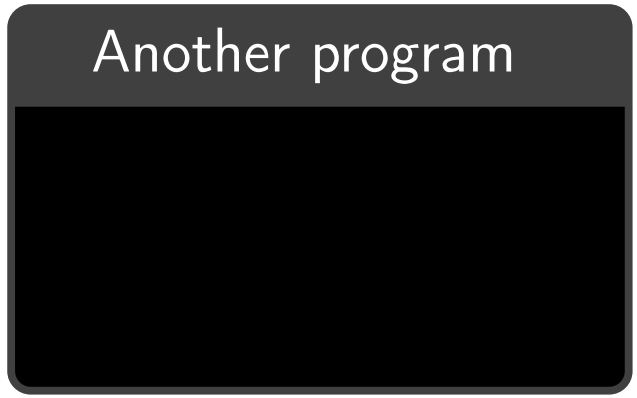

The context affects the behavior of the program.

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...
print(my_fun(n));
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(* terminates, prints 84 *)
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Testing programs in every possible context.

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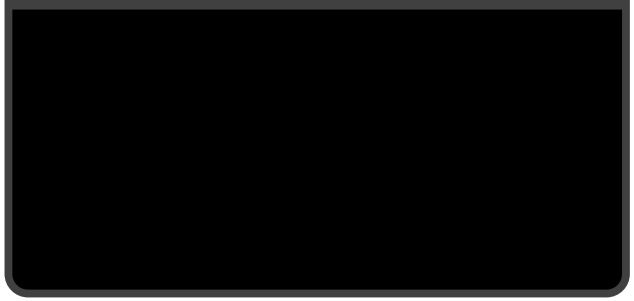

```
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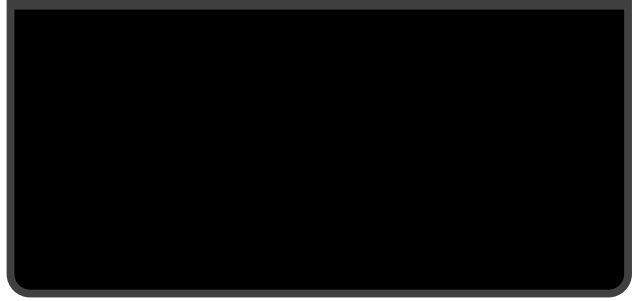

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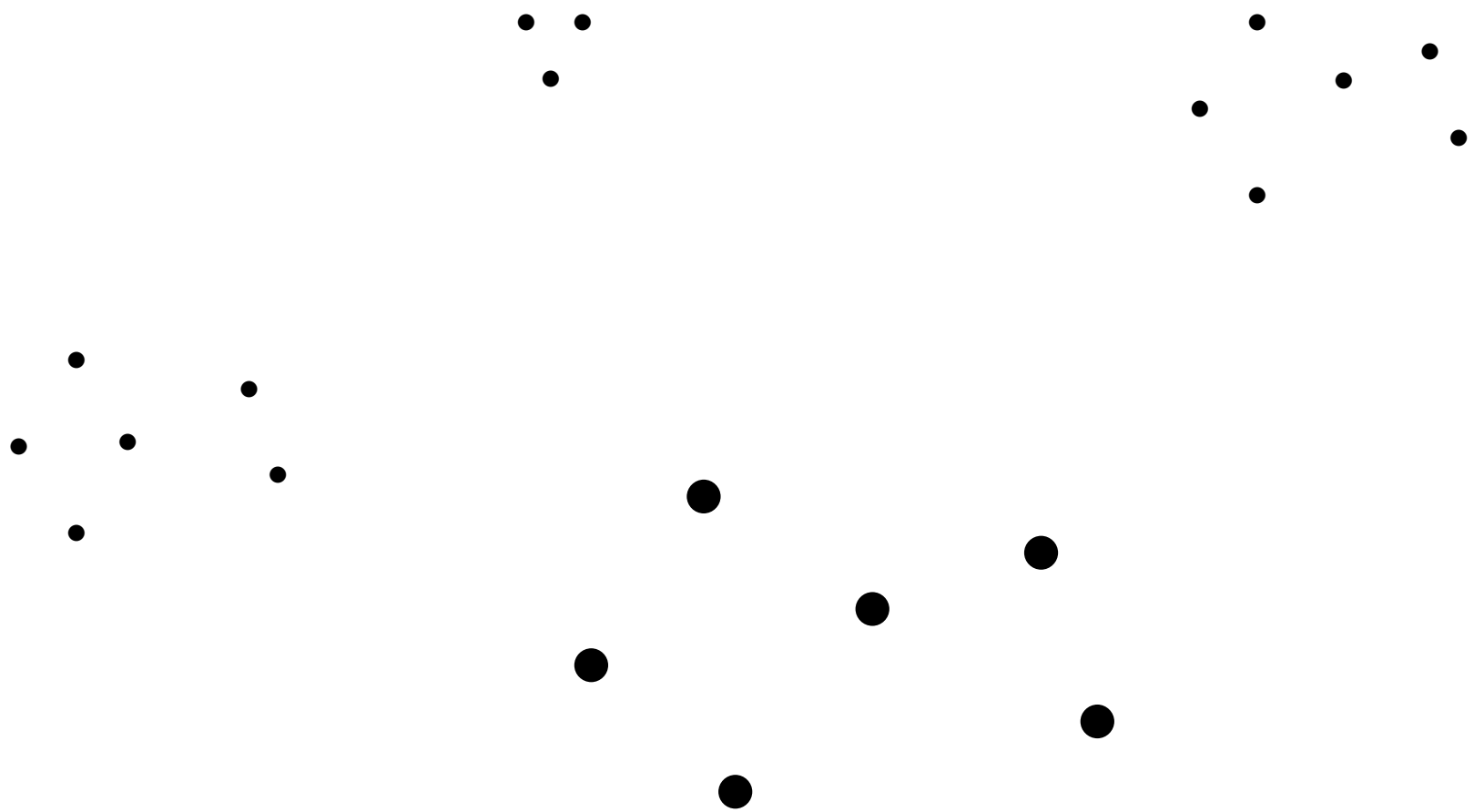
(In)Equational Theories

Program Equivalences in General:



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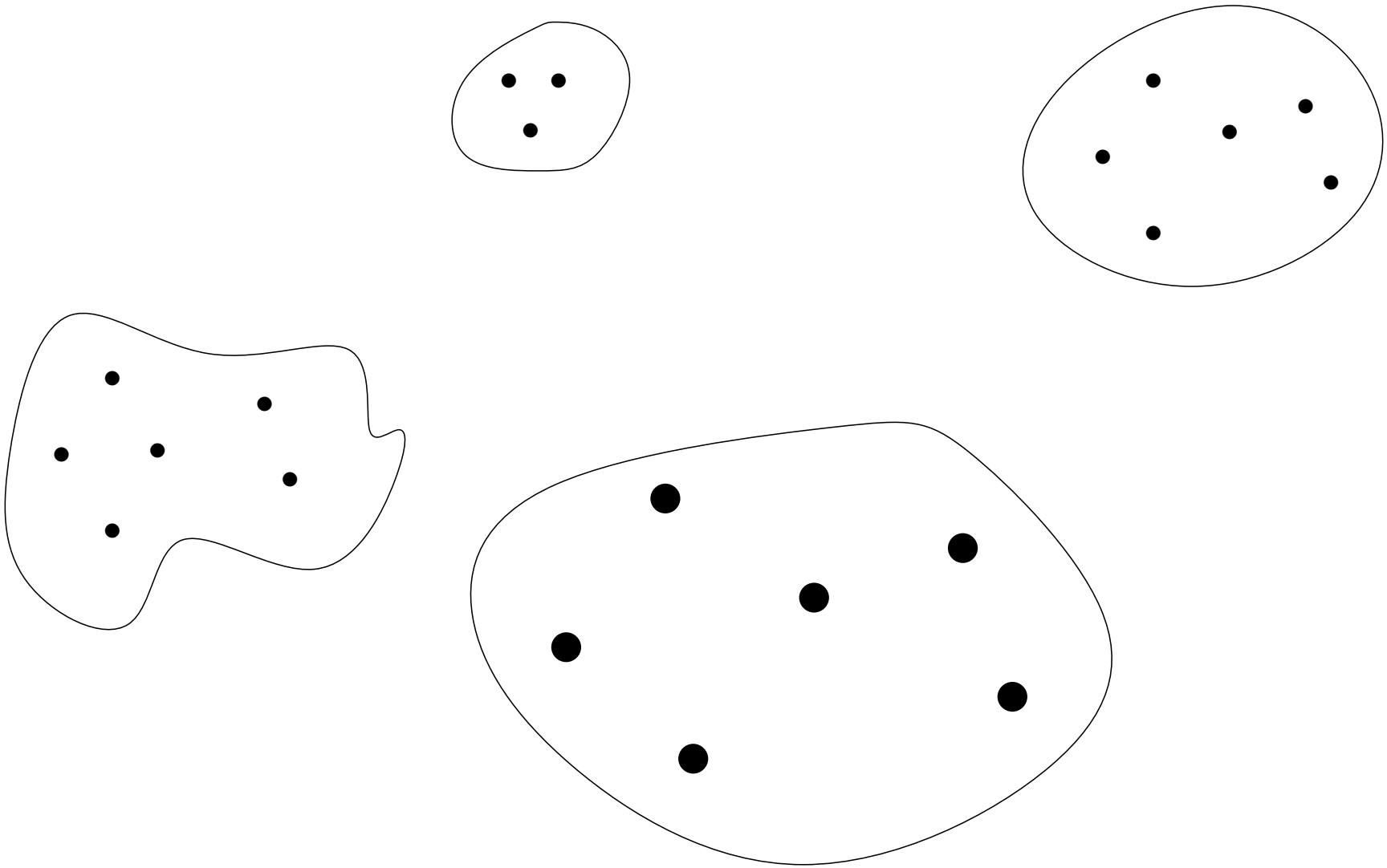

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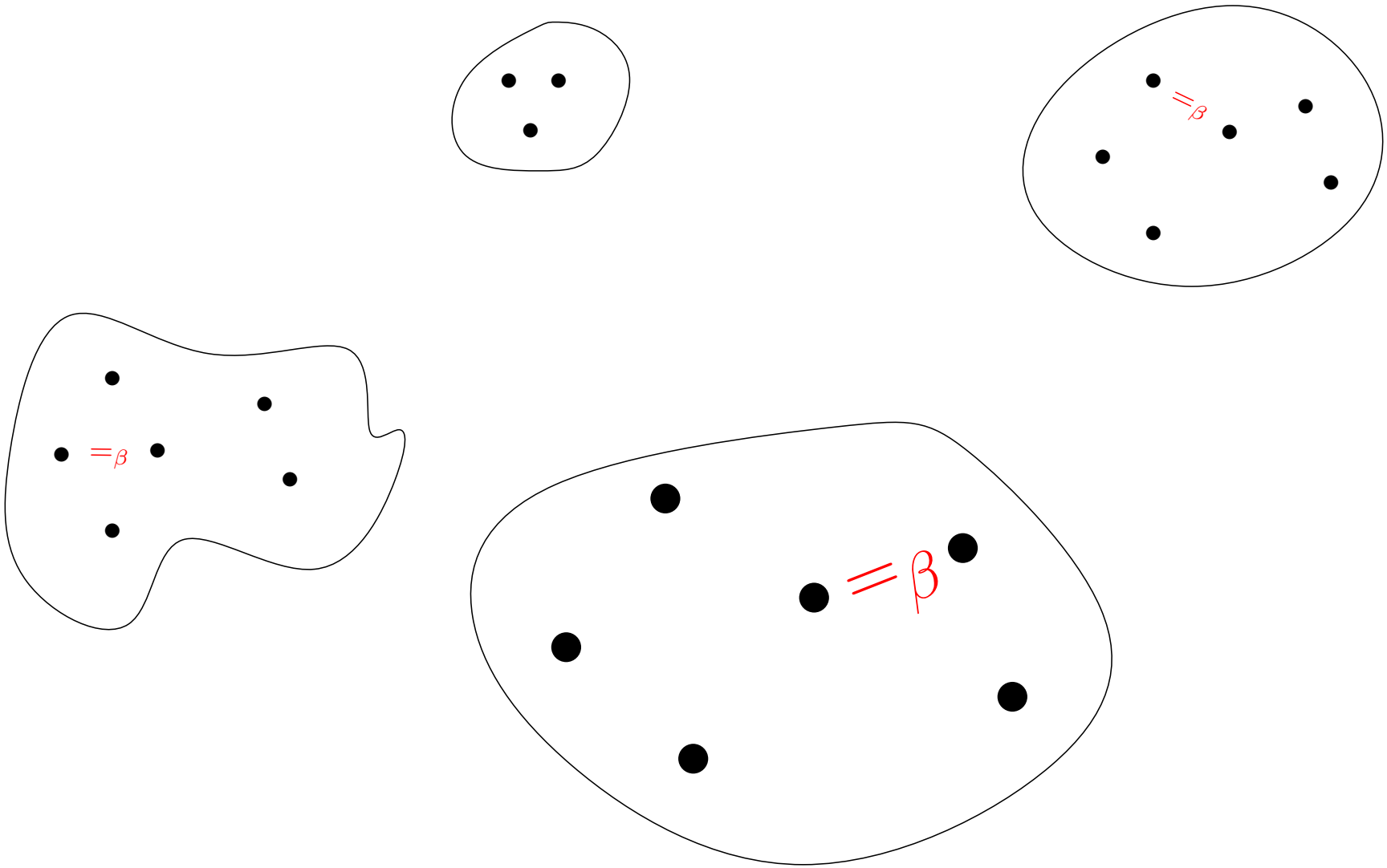

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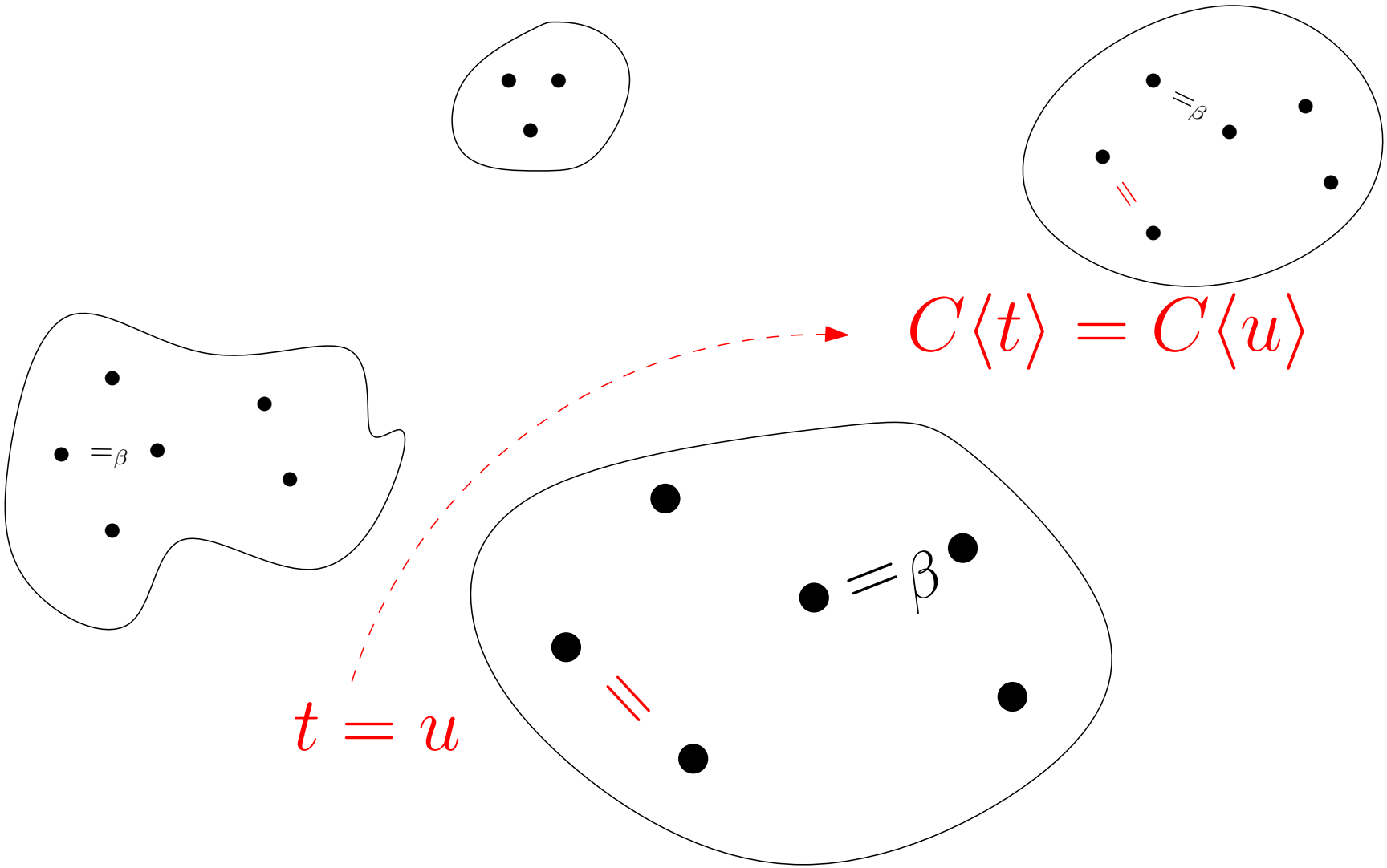

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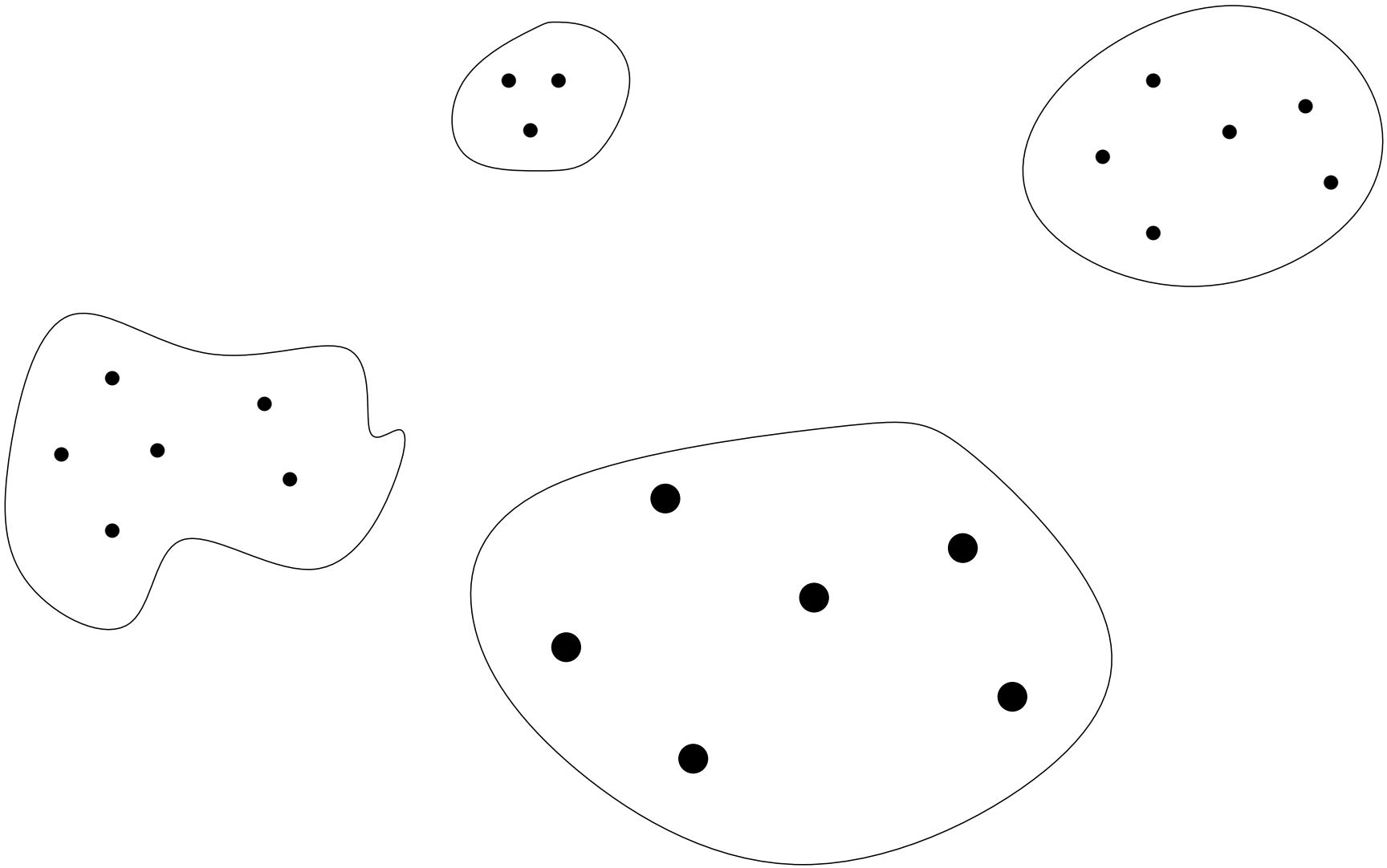

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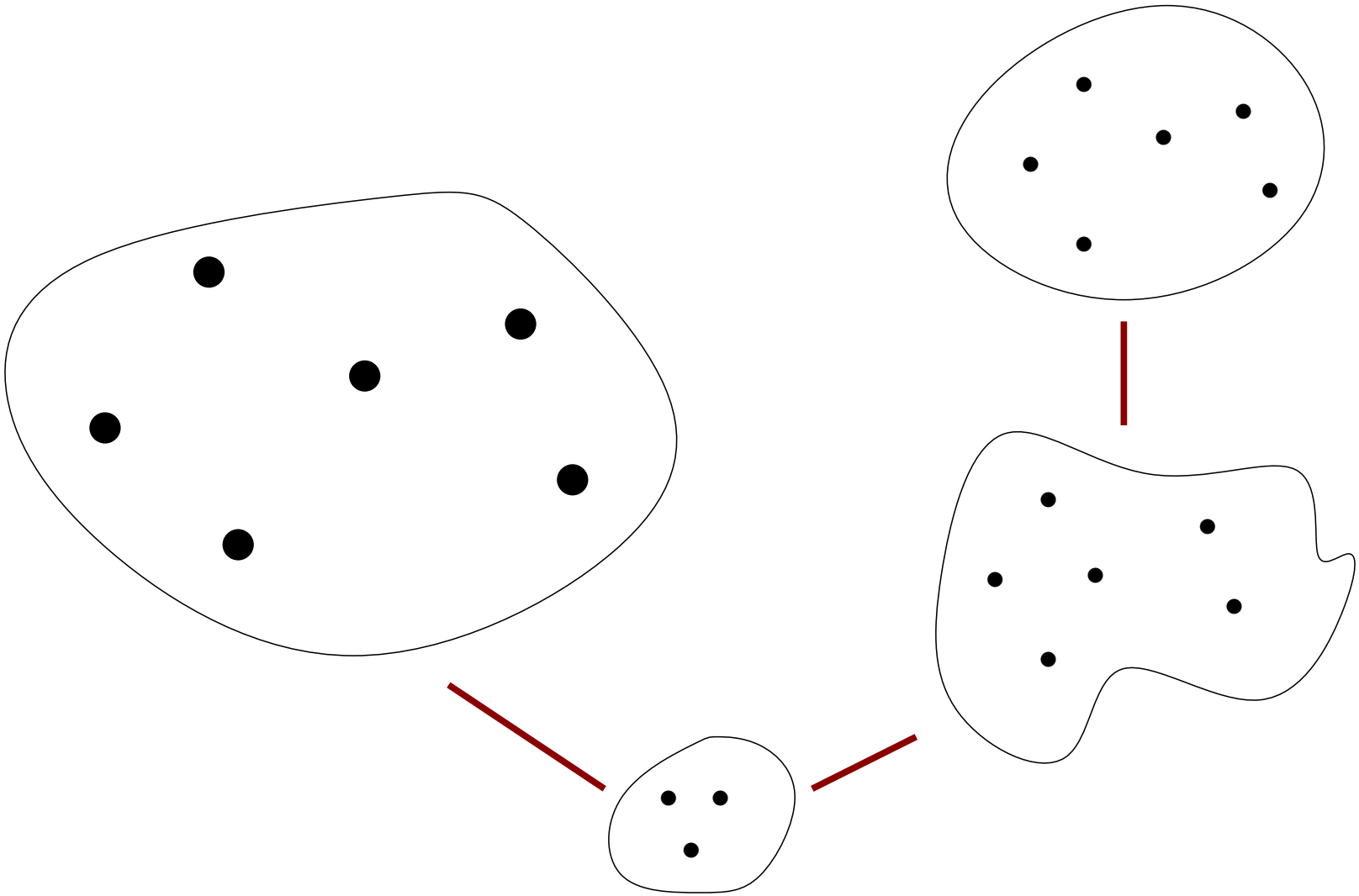

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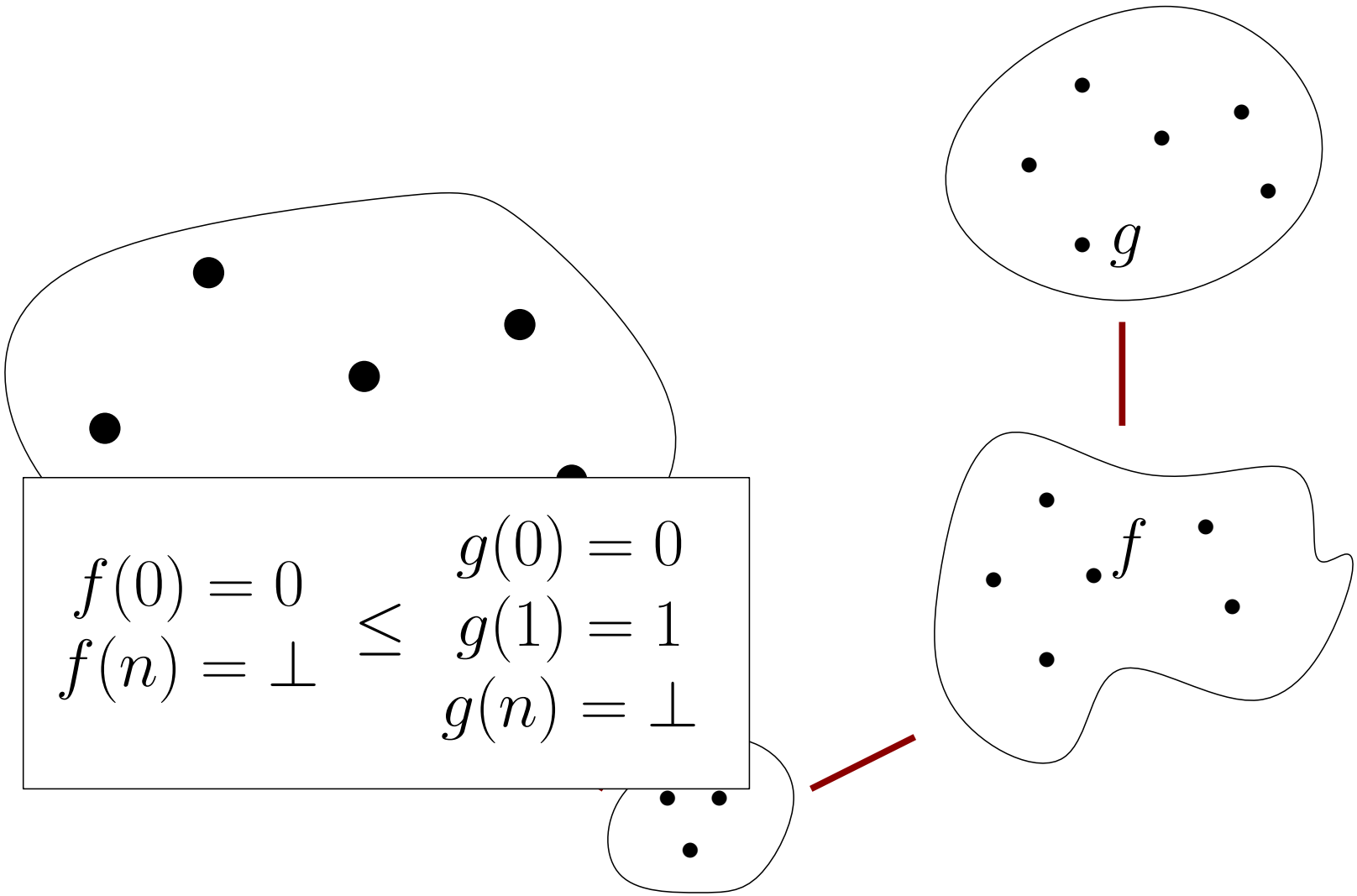

Which (In)Equational Theory, and for what?

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  a natural equational theory

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Refine **quantitatively** contextual equivalence, ignoring the internal behavior of programs!

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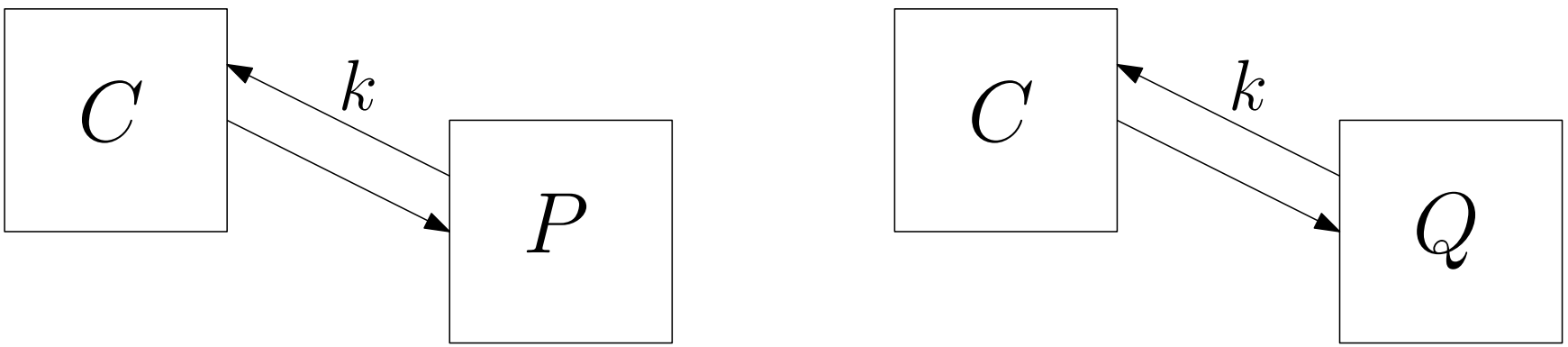

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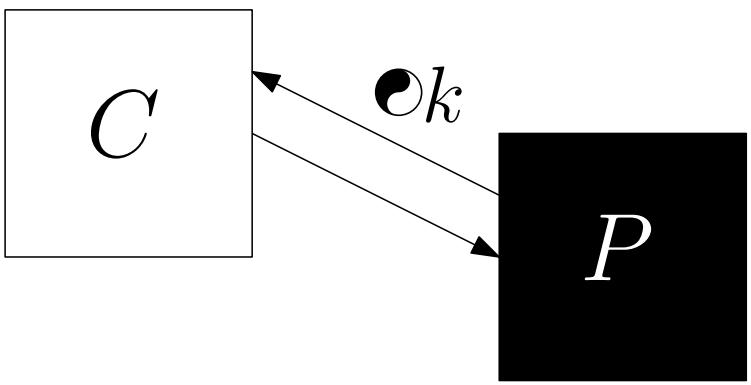

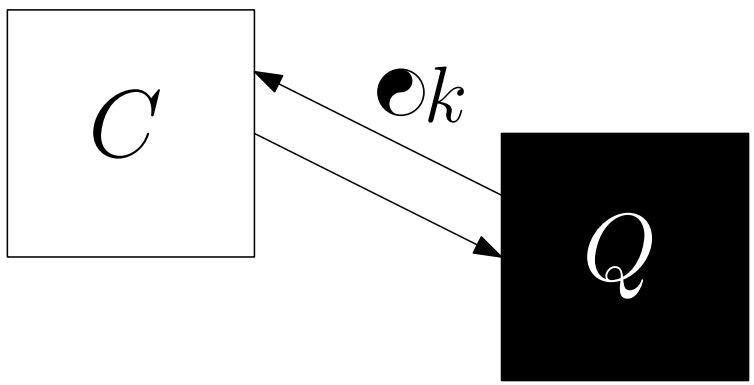

$$(\lambda_\bullet x.t) \bullet u \to_\tau t\{x \leftarrow u\}$$

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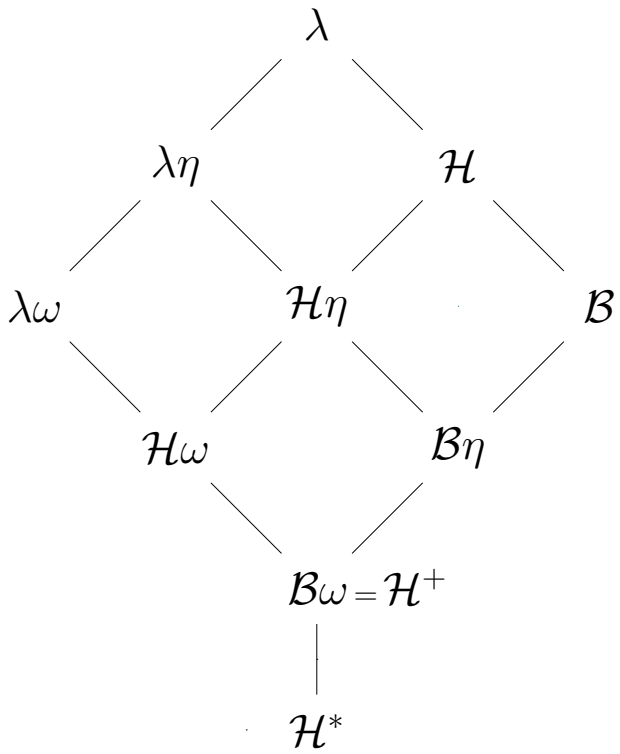
A Call-by-Value Odyssey



$(x + y) + (x + y) \quad = \quad \texttt{let } z = (x + y) \texttt{ in } z + z$

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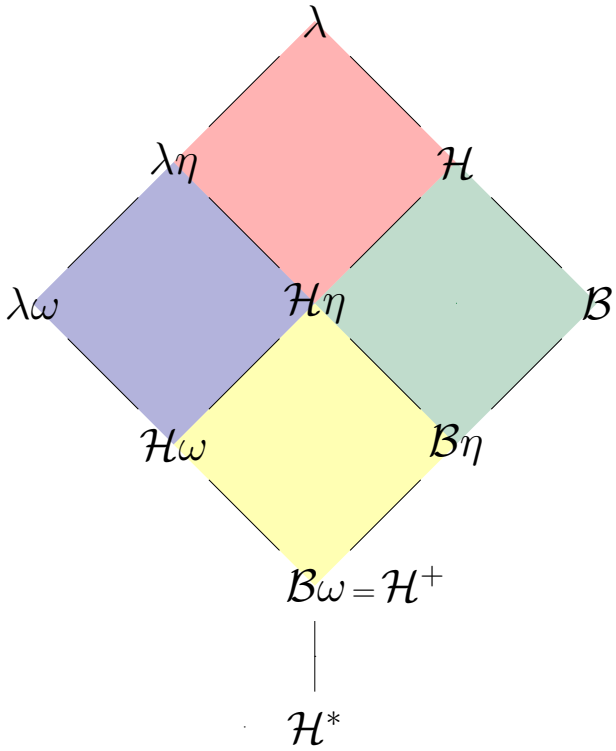

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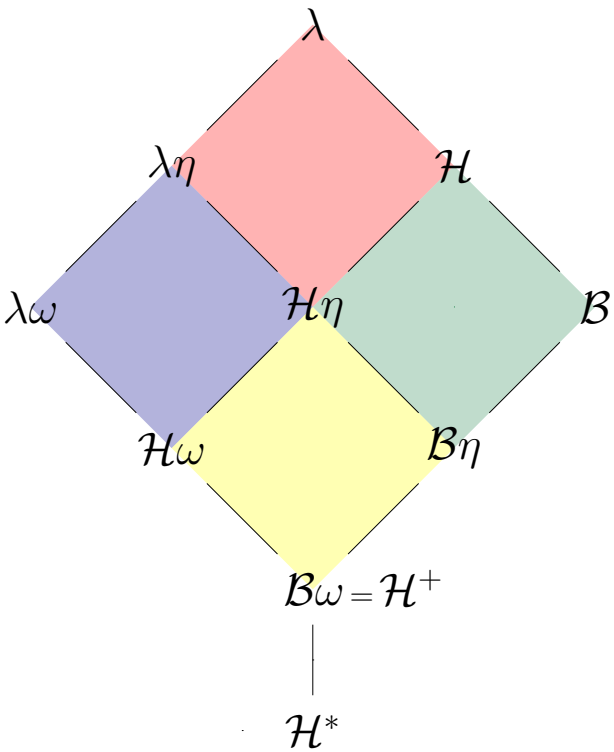

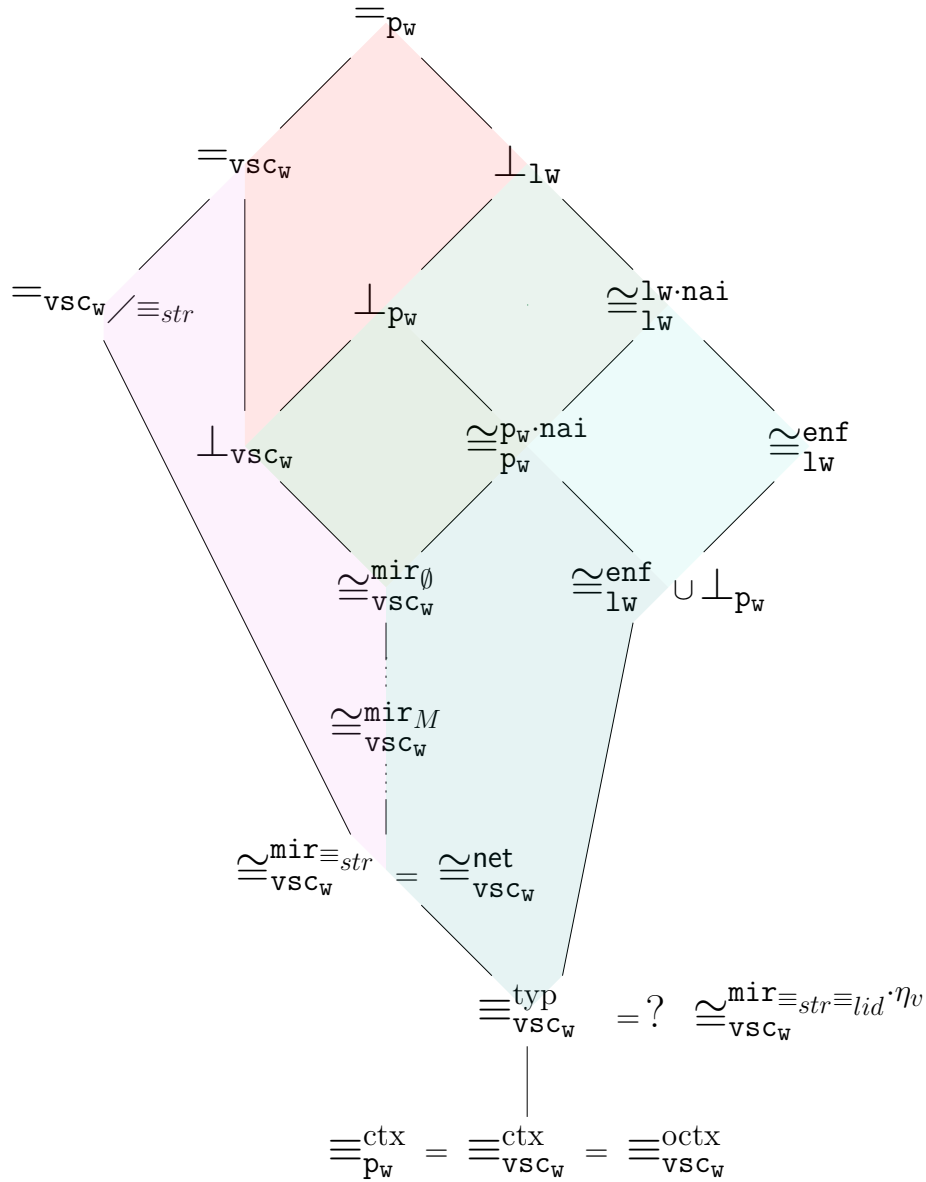

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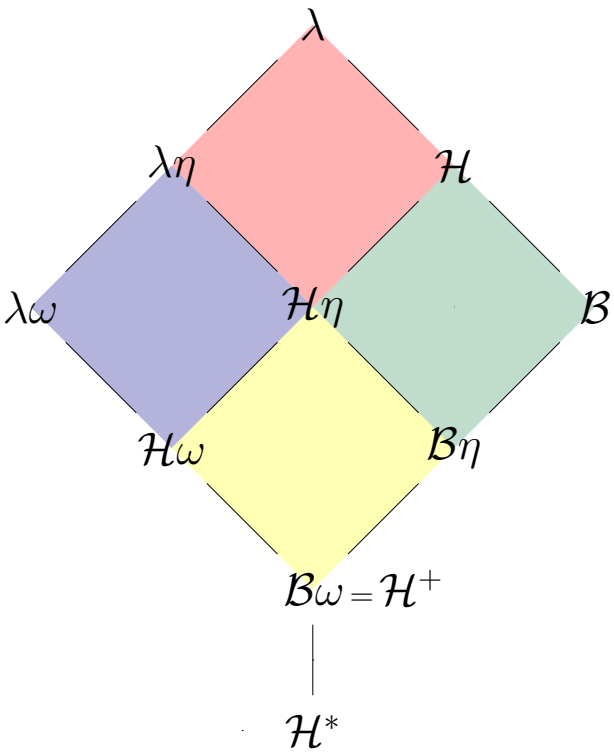

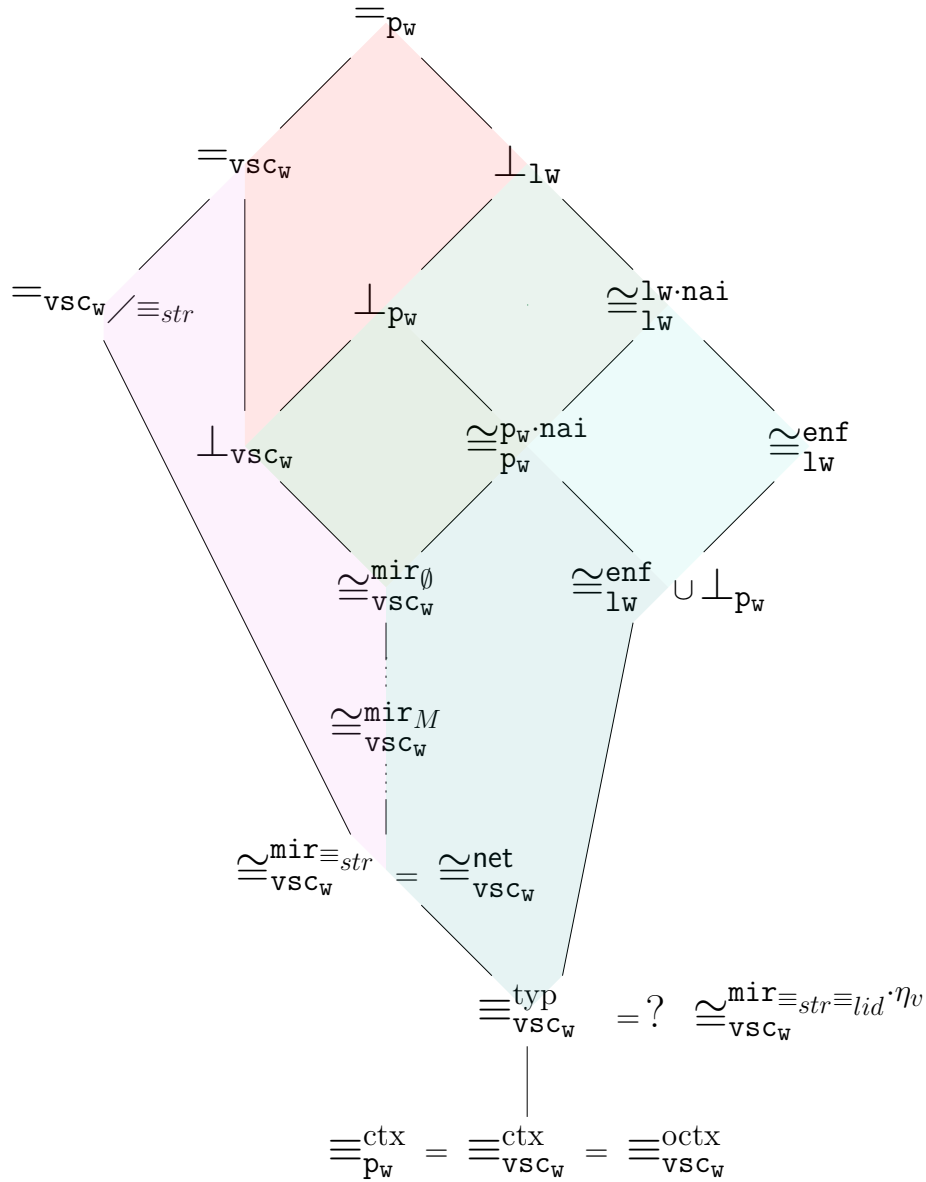

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Thank you!

Theory of Computation

Church's λ-Calculus

- A prototypical functional programming language

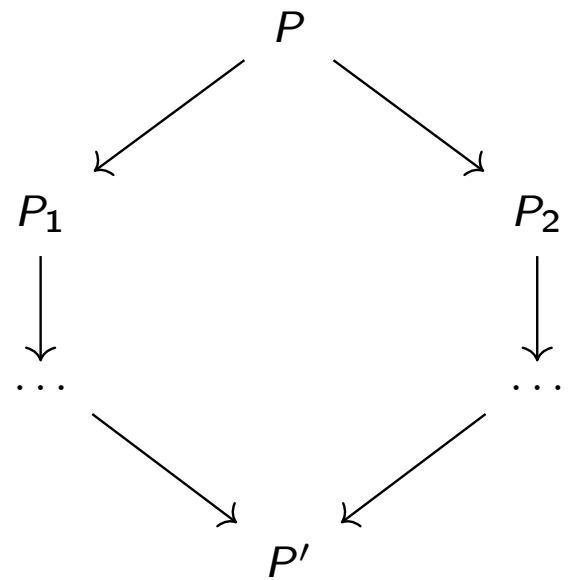
$x \mapsto f(x)$ is $\lambda x.f(x)$

β-reduction:

$(\lambda x.x+x)(n) \to_\beta n+n$

Rewriting Theory

- Study of Rewriting Sequences

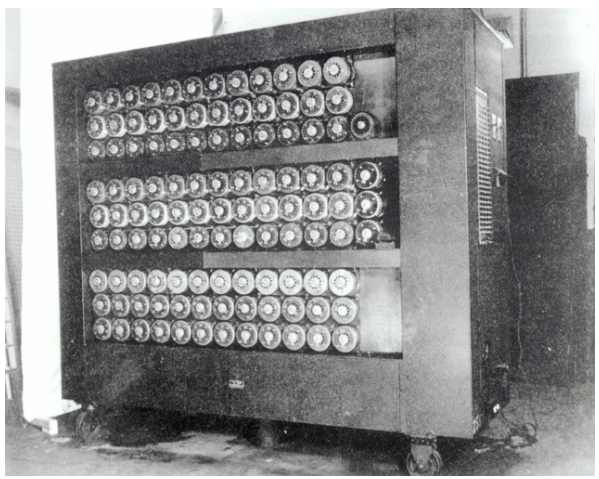


Semantics as Specification

Program equivalence can be studied through different **semantic** point of views.

Operational **Denotational**

Machine $\xleftarrow[\textit{interpreter}]{}$ Language $\xrightarrow{\textit{interpretation}}$ Abstract Math. Structures



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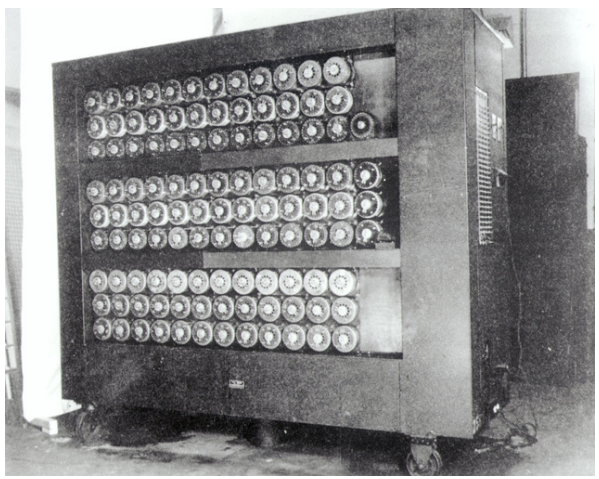

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Contextual Equivalence depends on the Evaluation Paradigm

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There are looping terms like $\Omega \to \Omega \to \Omega \to \cdots$
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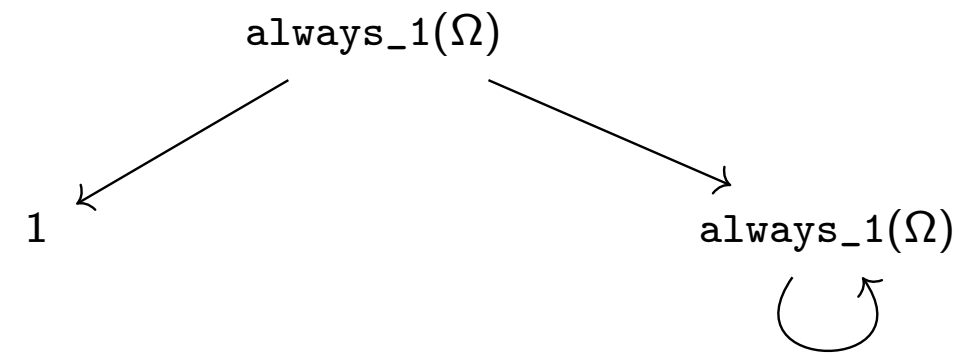
$$10 \xleftarrow{\text{CbN}} \texttt{double(2+3)} \xrightarrow{\text{CbV}} 10$$

There are looping terms like $\Omega \to \Omega \to \Omega \to \cdots$
Fix a program $\texttt{always\_1} \coloneqq \lambda x.1$. How to compute $\texttt{always\_1}(\Omega)$?

Call-by-Name

Call-by-Value



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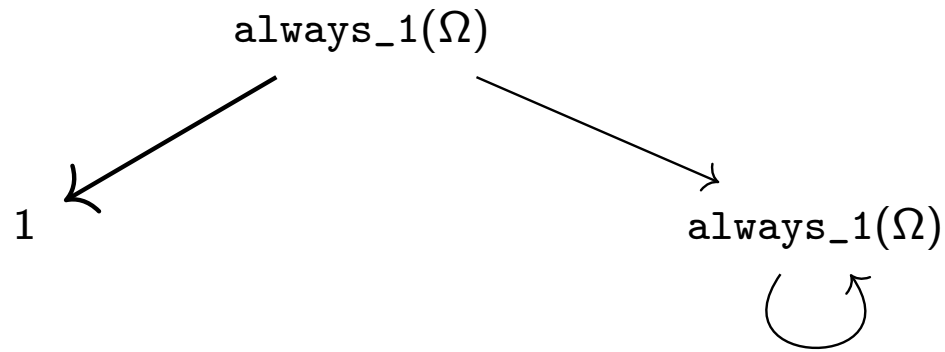

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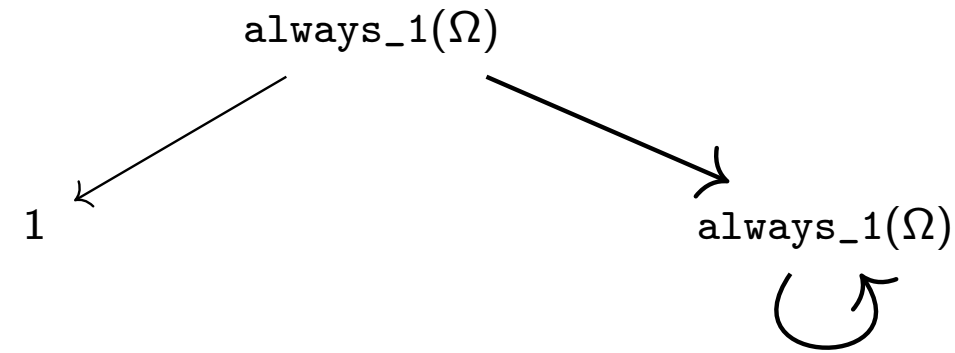

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Normal Form Bisimulations

How do we prove equivalence of (co)recursive programs?
Typically: equating fixpoint combinators

- **"Infinitely" Inspecting the Code:**

A program

```
let rec f :=
                  f(rec f).
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Another program

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let rec' f :=
              f(f(rec' f).
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An (infinitary) semantics of these programs: $\lambda \mathtt{f}.\mathtt{f}(\mathtt{f}(\mathtt{f}(\mathtt{f}\ldots)))$

Building and Comparing infinite structures is done by **coinduction**.

$$\mathrm{BT}(\mathtt{rec}) = \lambda f.f \quad = \mathrm{BT}(\mathtt{rec'})$$
$$\Big|$$
$$f$$
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(In)Equational Theories

Fix an evaluation strategy $\mathtt{s}$ within a language of terms $(\mathcal{L}, \rightarrow_{\mathcal{L}})$, $\rightarrow_{\mathtt{s}} \subseteq \rightarrow_{\mathcal{L}}$.

An inequational theory $=_{\mathcal{T}}$ for $\mathtt{s}$ is:

1. an *Equivalence Relation* on terms, $=_{\mathcal{T}} \subseteq \mathcal{L} \times \mathcal{L}$;
2. *Context-Closed*:
$$\text{if } t =_{\mathcal{T}} u \text{ then } \forall C,\ C\langle t \rangle =_{\mathcal{T}} C\langle u \rangle;$$
3. *Invariant under Computation*:
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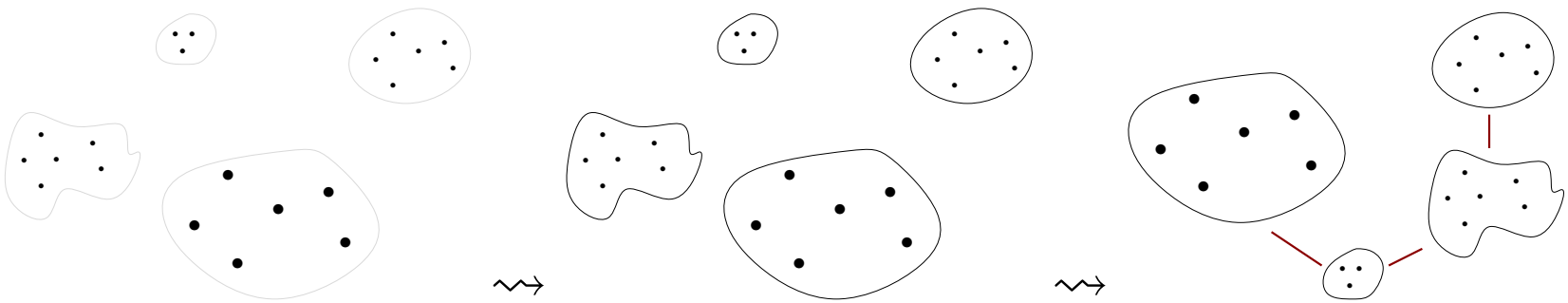
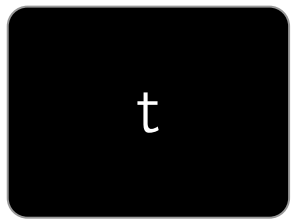
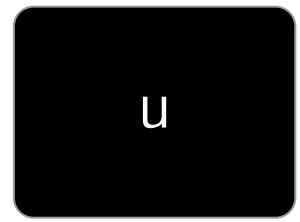


Contextual Equivalence and Preorder





When are two λ-terms equivalent? [Mor68]

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The s-contextual preorder is generally an inequational theory.

Sufficient conditions on s and $\mathcal{L}$ are provided in the thesis.

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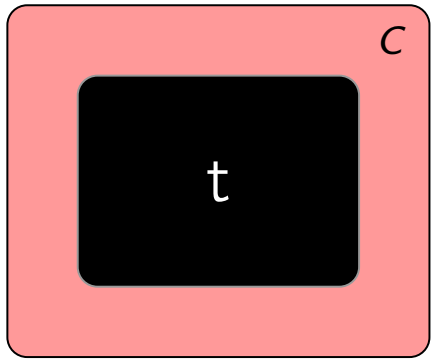

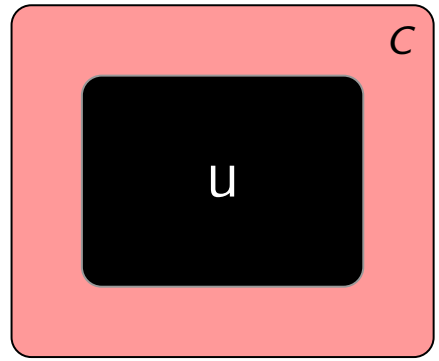

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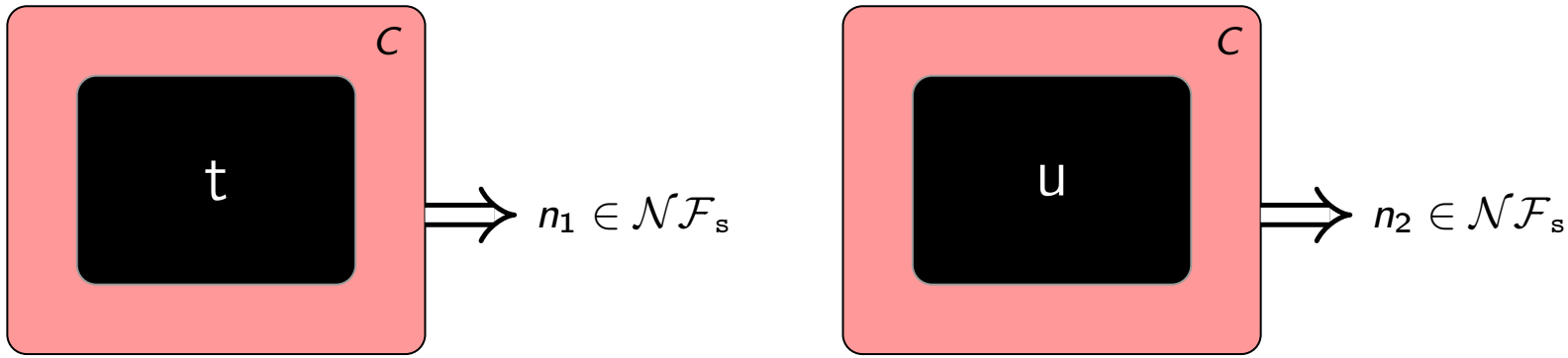

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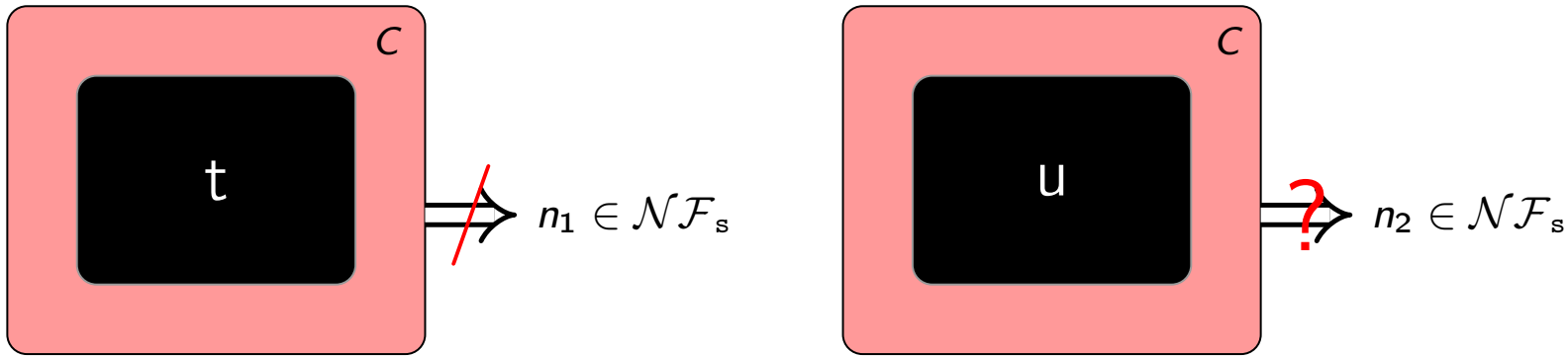

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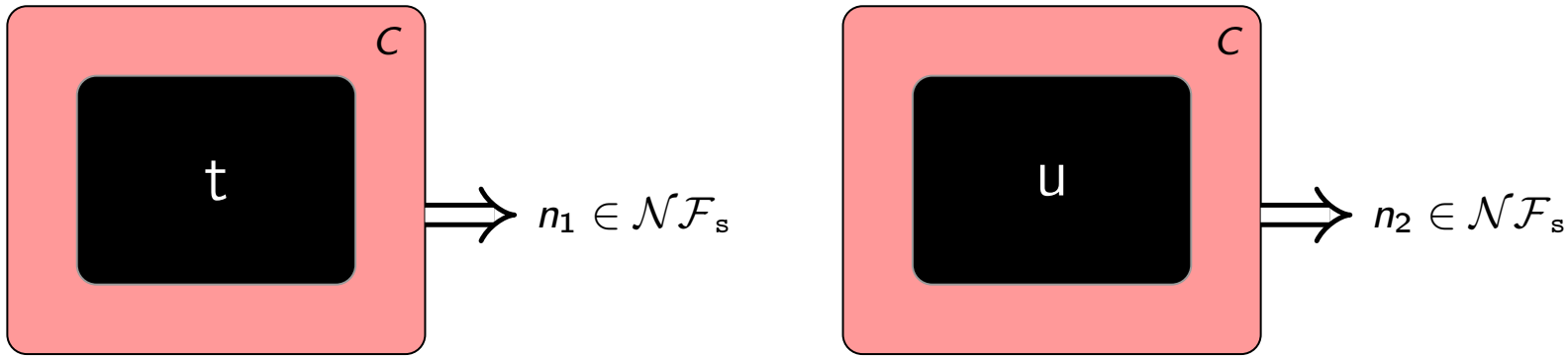

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Part I: Overview of Program Equivalences

Definable Type Equivalence
Operational Game Semantics
Weak Head Ineq. Theories
Interaction Improvement

Inequational Theories 🗋 FoSSaCs 2024

Light and Co Genericity

Maximality

Formalized CbN development 🗋 ITP 2025, formalized in Abella

Improvement theories

Normal form bisimulations
(and Böhm trees)
Applicative bisimulations

Type Equivalence (for CbN)

Part II:
A Pilgrimage into CbN

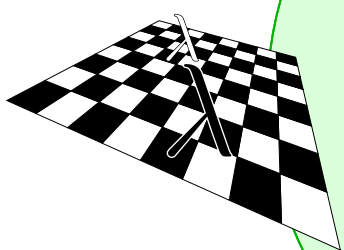
🗋 POPL 2025

Interaction Equivalence

Checkers Calculus

Ctx. Characterization of
Böhm tree equivalence

Checkers Relational Semantics



🗋 TYPES
PostProc 2025

Type Equivalence for lazy CbN

Part III:
A CbV Odyssey

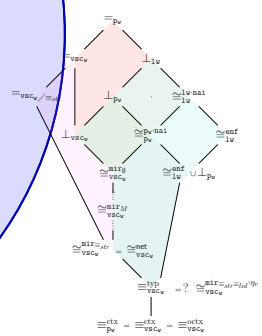
Theory of Call-by-Value
Light and Co CbV Genericity
Call-by-Silly

🗋 FoSSaCs 2024

🗋 FSCD 2024

Various CbV Normal form bisimulations

Type Equivalence for CbV



🗋 TechRep 2024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